

HB 1 test

20 Nov In class Test
90 min 50%

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J period 50% in exam hall

Part A

Formal language and model of
computation

Part B

Limits of computing

Introduction to language

Language is defined as a set of symbols -

Language is defined as a set of symbols
alphabet which are combined into
acceptable strings

The rules which tell us how to combine
strings in a sensible way is called grammar

Alphabet + string + grammar = Language

Alphabet -

A finite, non-empty set of symbols is
called an alphabet

Eg $\Sigma = \{a, b, c\}$

String -

A finite sequence of symbols from the
alphabet (placed next to each other)

Eg aa, abc, ccb are strings in Σ .

Language -

If Σ is alphabet, then a language over
 Σ is a set of strings whose symbols are

E.g. - If $\Sigma = \{a, b\}$ then $L = \{ab, aab, abbbaa\}$ is an example of a language over Σ .

If Σ is an alphabet, then Σ^* denotes the infinite set of strings that are made up from Σ . Including empty set

- Examples of languages over an alphabet Σ are the set:

- $\emptyset$, $\{\lambda\}$, Σ , and Σ^* .

Languages are set of strings, so they can be described by the usual set operations of union and intersection

Product of two languages

If L and M are languages, then the new language called the product of L and M is defined as

$L \cdot M$ (or only L and M)

0

$$L \cdot \{\lambda\} = L$$

$$L \cdot \emptyset = \emptyset$$

Operation of concatenation is **Not**
Commutative. Order does matter

Concatenation is associative.

If L , M and N are languages, then

$$L \cdot (M \cdot N) = (L \cdot M) \cdot N$$

If L is a language, then the product
 $L \cdot L$ is denoted by L^2 .

$$L^0 = \{\lambda\}$$

$$L^n = L \cdot L^{n-1}, \text{ if } n > 0$$

If L is a language over Σ then
the closure of L is the language

denoted by L^* and is defined as follows

$$L^* = L^0 \cup L^1 \cup L^2 \cup \dots$$

The positive closure of L is signified with L^+

$$L^+ = L^1 \cup L^2 \cup L^3 \cup \dots$$

↖ ↗
No empty string

Only difference is empty string

In CS, a language is a set of strings

Grammars

Two collections of symbols are necessary

- **Terminals** are those symbols which

all strings in language be made

- **Non Terminals** are temporary symbols
Used to define the grammar replacement
rules.
Must be replaced by terminals.

$$\alpha \rightarrow \beta$$

" replace α by β

If $\Sigma = \{a, b\}$ and S is a non-terminal
symbol then the rules
 $S \Rightarrow aS, S \Rightarrow \lambda$ are examples
of productions for a grammar L .

Example

Grammar is defined by

- set of terminals $T = \{a, b\}$
Only non-terminal start symbol S
Set of productions

Start of grammar rules.

$$S \rightarrow \lambda, S \rightarrow aSb$$

or

$$S \rightarrow \lambda \mid aSb$$

$$S \rightarrow aSb$$

$$S \rightarrow a a S b b$$

$$S \rightarrow a a b b$$

← Replaces S with aSb

⇒ Replaces S with λ

Every grammar must have a start symbol.

A string made up from both terminals and non-terminals is called **sentential form**

Derivation
 $a \rightarrow B$

$$xay \Rightarrow xby$$

$$\underline{L(G)} = \{s \mid s \in T^* \text{ and } S \Rightarrow^+ s\}$$

A production is **recursive** if its left side occurs on its right side

$$S \rightarrow aS \text{ is recursive}$$

Grammar for an infinite language must be directly or indirectly recursive.

Grammar is unique. A language can have many grammars.

Example not

$$S \rightarrow aS \mid bS$$

$$L(G) \neq \emptyset$$

because there will always be
non-terminals